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On the covariant quantization of tensionless bosonic strings in AdS spacetime

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Abstract: The covariant quantization of the tensionless free bosonic (open and closed) strings in AdS spaces is obtained. This is done by representing the AdS space as an hyperboloid in a flat auxiliary space and by studying the resulting string constrained hamiltonian system in the tensionless limit. It turns out that the constraint algebra simplifies in the tensionless case in such a way that the closed BRST quantization can be formulated and the theory admits then an explicit covariant quantization scheme. This holds for any value of the dimension of the AdS space.

Keywords: Bosonic Strings, BRST Quantization, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

The tensionless limit of string theory is as important to be understood as the field theory of massless particle excitations is. That's because, as in field theory the most useful and beautiful symmetries, namely gauge symmetries, are typical of massless particles field theories, we expect higher bigger symmetries and nicer quantum properties to appear in the tensionless limit of string theory too [1].

As far as the flat background space is concerned, one finds in fact more than just new gauge symmetries. In [2] the disappearance of the very concept of the critical dimension was noticed.[1] That result made clear that the need of fully solving the Liouville theory to analyze the strings in arbitrary space-time dimension [4] is specific and peculiar of the tensile string. This happens since in the tensionless limit the conformal anomaly itself gets scaled away. The link between the tensionless limit of string theory in flat background and higher spin field theories was also explored in [5]. Moreover, in [6], the arising of a new infinite symmetry has been explicitly obtained and the tensionless limit of the interacting second quantized string was analyzed.

In this notes we start studying the problem of the tensionless limit of strings on the simplest negatively curved spaces, namely AdS spaces. Let us notice that, since the expansion of the string on curved backgrounds has been mostly studied in the point-like regime $\alpha' \to 0$, we are facing an almost unexplored subject[2] and the existing expansion methods are therefore useless in the $\alpha' \to \infty$ limit.

[1] Let us recall also the papers [3] where a similar, but still less systematic, approach was sketched.

[2] The approach we take is different to the "Null String"s one by Schild [7] to which the expansion technique developed first in [8] applies. The basic difference is that in the "Null String" approach one takes the tensionless limit at fixed σ-model variables, while we keep fixed the string oscillator variables in order to have an a priori control on the mass spectrum.

Our approach is quite simple and maybe elementary. It follows by considering the AdS space an hyperboloid in an higher dimensional flat auxiliary space. In such a picture we implement the restriction to the hyperboloid as a lagrangean constraint to the free theory in the ambient space. The natural set-up for the analysis of such a problem turns out then to be the constrained hamiltonian formalism.[3] It turns out that the constraint algebra structure simplifies in the massless/tensionless case in such a way that the arising of a larger gauge symmetry takes place. More specifically, it happens that the geometric constraints are second class for generic values of the tension parameter, but in the massless/tensionless regime it is possible to single out one half of them which, together with the reparametrization/Virasoro generators, are first class. This property opens the way to a well defined BRST covariant quantization of the system which we develop here in detail.

In few words, the main point is that massless free excitations on AdS can not probe the strength of the space-time curvature and therefore, in this case, the assignment of the value of a finite AdS radius should be regarded as a gauge choice (the assignment of a zero radius being a degenerate gauge fixing condition). Let us underline again that this property is special of massless/tensionless excitations on AdS space-time.

Let us notice that such a phenomenon is linked to the very definition of the masslessness itself in AdS space (see [11]) and could be rephrased, from the second quantized point of view, by analyzing the theory in terms of higher spin fields. In such terms, it should correspond to the phenomenon noticed in [12], i.e. special slope values at which extra gauge degrees of freedom arise in higher spin field theories.

Our main result is then a covariant quantization scheme for tensionless strings in AdS where no restriction to the space-time dimension appears in the form of a critical dimension, the constraint algebra being a Lie algebra without any non trivial central extensions.

In the following first warm up section 2, we will treat the easy case of the scalar free massless particle in AdS in order to clearly explain the relevant procedure. In the subsequent section 3 we study the tensionless limit of the free open and closed bosonic string in AdS space by extending the method to such a more interesting cases. A final section 4 points out some open questions and possible further developments for the second quantized tensionless string theory on AdS background as a theory of interacting higher spin fields and AdS/CFT at null CFT coupling.

2. Free spinless bosonic massless particles in AdS

Let us model the dimension d AdS space as an hyperboloid in a flat $d+1$ dimensional space. Labeling the coordinates in $\mathbb{R}^{d+1}$ as x^μ, as $\mu = 0, \ldots, d$, the embedding equation is simply

$$x^\mu \eta_{\mu\nu} x^\nu = -x_0^2 + x_1^2 + \cdots + x_{d-1}^2 - x_d^2 = R^2$$

which defines the quadratic form η. We will usually write x^2 for $x^\mu \eta_{\mu\nu} x^\nu$ and $uv = u^\mu \eta_{\mu\nu} v^\nu$ for various (co)vectors.

[3] The covariant quantization of the bosonic string in flat spacetime was originally obtained in [9]. For a review of string theory in flat space as a constrained hamiltonian system, see [10].

The system of a spinless bosonic massless particles in AdS is most symmetrically described in the hamiltonian formalism. Actually we can study it as a constrained hamiltonian system of a free massless bosonic particle in $\mathbb{R}^{d+1}$ constrained to the AdS hyperboloid of dimension d. The natural set of constraints (which follow from the Dirac consistency procedure applied to the massive free particle with lagrangean constraint $x^2 = R^2$ and by the subsequent zero mass limit and the removal of the Lagrange multiplier) are

$$p^2 = 0\,, \qquad x^2 - R^2 = 0 \qquad \text{and} \qquad xp = 0 \tag{2.1}$$

which, together with a further non-singular gauge choice — for example a light cone gauge $vx = \tau$, with τ the hamiltonian time coordinate — give a good hamiltonian formulation of the system, i.e. form a non degenerate set of constraints. Differently from the massive case, we can single out from the above set a smaller one of first class constraints, namely $p^2 = 0$ and $xp = 0$ whose Poisson bracket algebra is closed. This shows that the spinless massless free particle on AdS is aware of being on a constant negatively curved space, but is not able to feel the strength of such a curvature. Put in another way, the assignment of the AdS radius can be viewed as a gauge fixing condition for the system.

As far as the quantization is concerned, we promote the canonical pair (x^μ, p_μ) to operators obeying the Heisenberg algebra $[x^\mu, p_\nu] = i\delta^\mu_\nu$ as well as the constraint functions to the hermitean operators p^2 and $\frac{1}{2}(xp + px)$. Notice that the first class constraint algebra still closes, namely[4] $[p^2, \frac{1}{2}(xp + px)] = -i2p^2$. This enables us to perform the Dirac quantization of the dynamical system, that is to formulate the consistent wave equation system

$$p^2\psi = 0 \qquad \text{and} \qquad \frac{1}{2}(xp + px)\psi = 0\,. \tag{2.2}$$

Choosing hyperbolic polar coordinates in $\mathbb{R}^{d+1}$ one can solve the radial dependence of the wave function by the second of eqs. (2.2) and stay with a single resulting wave equations in AdS intrinsic coordinates. This fixes unambiguously the scalar massless field curvature coupling in AdS.

There are in fact several possible generalizations of the above constraint algebra (for example one can add the coupling with a constant curvature bulk electromagnetic field) and these could be interesting constrained dynamical systems to study.

It is also possible rephrase the quantization procedure at a BRST level. This is done by introducing the independent anticommuting ghost real pairs (c, b) and (c', b') and encoding the constraint algebra in the BRST charge

$$\mathcal{Q} = \frac{1}{2}cp^2 + c'\left[\frac{1}{2}(xp + px) - i(cb - bc)\right] \qquad \text{obeying} \qquad \mathcal{Q}^2 = 0\,,$$

where the antisymmetric ordering for the real ghosts has been applied. The anti-hermitean

[4]Notice that if a mass term would be added as $p^2 \to p^2 + m^2$ then such a property would not be true anymore and a more elaborate scheme, from the point of view of the constrained system analysis, should be applied.

ghost number operator is $\mathcal{G} = \frac{1}{2}[c, b] + \frac{1}{2}[c', b']$ and is such that $[\mathcal{G}, \mathcal{Q}] = \mathcal{Q}$. The BRST state cohomology $Ker\mathcal{Q}/Im\mathcal{Q}$ can be easily calculated to reproduce[5] eq. (2.2).

3. Tensionless bosonic free strings in AdS

After the previous warm up section, let us now enter the main subject of the present note, namely the tensionless limit of free bosonic strings in AdS space. We will follow the same strategy developed in the previous section for the spinless scalar particle and generalize it to open and closed free bosonic strings.

3.1 Open strings

3.1.1 The constraint algebra

In this subsection we analyze the constrained system of open strings in $\mathbb{R}^{d+1}$ bound to stay on the hyperboloid $x^2 = R^2$ in the tensionless limit. The study of this system amounts to the extraction of the leading order terms in α' out of the Virasoro constraints

$$\frac{1}{2}\left[2\pi\alpha' P^2 + \frac{1}{2\pi\alpha'}(X')^2\right] \qquad \text{and} \qquad X'P \tag{3.1}$$

and the geometric constraints[6]

$$\frac{1}{\alpha'}\left(X^2 - R^2\right) \qquad \text{and} \qquad XP\,. \tag{3.2}$$

Then, following the method explained in the previous section, we have to select out of the geometric constraints a subset such that, together with the contracted Virasoro constraints, one gets a closed algebra under commutation so that one is left with first class constraints only and a BRST quantization procedure is available. Let us notice that such a procedure was available for massless particles only on AdS and is respectively available for the tensionless strings only in AdS (while it is not for the tensile case when the subleading terms have to be kept). Moreover, since first class constraints weight twice — in the degrees of freedom counting — than second class ones (see for a review [13]), to get the correct degrees of freedom counting, the subset of constraints that we have to single out has to contain one half of the original geometric ones.

Before proceeding, let us now just fix some notation. The system is given by the string center of mass variables (x_μ, p^μ) and the infinite set of oscillators $(a_{n\mu}, a^*_{n\mu})$, with $n > 0$. They satisfy the usual canonical commutation relations (CCRs)

$$[x_\mu, p^\nu] = i\delta_\mu^\nu\,, \qquad [a_{n\mu}, a^*_{m\nu}] = \eta_{\mu\nu}\delta_{nm} \tag{3.3}$$

[5]Assuming standard partial invertibility for the relevant operators which generalizes the usual ones holding for the free massless relativistic particle in flat space.

[6]These can be shown to follow from the Dirac consistency procedure applied to the string lagrangian in $\mathbb{R}^{d+1}$ augmented by the lagrangian constraint $X^2 - R^2 = 0$. The Lagrange multiplier is already gauge fixed.

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and the other commutators are vanishing. The string coordinate and its conjugate momentum are ($\sigma \in [0, \pi]$)

$$
\begin{aligned}
X_\mu(\sigma) &= x_\mu + \sum_{n>0} \sqrt{\frac{2\alpha'}{n}} \left(a_{n\mu} + a^*_{n\mu}\right) \cos(n\sigma) \,, \\
P^\mu(\sigma) &= \frac{p^\mu}{\pi} + \frac{1}{i\pi} \sum_{n>0} \sqrt{\frac{n}{2\alpha'}} \left(a_{n\mu} - a^*_{n\mu}\right) \cos(n\sigma).
\end{aligned}
$$

Let us notice that we define the string expansion in the flat auxiliary space as the free string expansion. This is the only correct possibility once we required that the tension parameter enters the oscillator expansion of the string canonical variables independently on the further constraints fixing the curved space the string will be constrained on, that is independently on the geometric constraints.

The extraction of the leading order from the Virasoro constraints (3.1) is exactly equal to the calculation in the flat space (see [2, 6]), i.e. we have (after rescaling)

$$
L_0 = p^2 \,, \qquad L_n = p \cdot a_n \qquad \text{and} \qquad L^*_n = p \cdot a^*_n \,.
$$

A choice of one half constraints out of the geometrical ones (3.2) is to select the zeromode part of XP and the oscillating modes of $\frac{1}{\alpha'}\left(X^2 - R^2\right)$, that is

$$
D_0 = \int_0^\pi d\sigma : XP : (\sigma) = \frac{1}{2}(xp + px) + \frac{1}{2i} \sum_{n>0} \left(a_n^2 - a^{*2}_n\right)
$$

and the leading order coefficients in the Fourier expansion of $\frac{1}{\alpha'} : XX' : (\sigma) = \sum_{p>0} \sin(p\sigma) \times (D_p + o(1/\sqrt{\alpha'}))$, namely

$$
\begin{aligned}
D_p = &\sum_{n>0} \left(\sqrt{\frac{p+n}{n}} - \sqrt{\frac{n}{p+n}}\right) \left(a_{p+n} a_n + a^*_{p+n} a_n + a^*_n a_{p+n} + a^*_{p+n} a^*_n\right) + \\
&+ \sum_{m=1}^{p-1} \sqrt{\frac{m}{p-m}} \left(a_m a_{p-m} + a^*_{p-m} a_m + a^*_m a_{p-m} + a^*_{p-m} a^*_m\right) ,
\end{aligned}
$$

where p runs over the positive integers (and the second sum is not there for $p = 1$). Notice that D_0 and D_p are hermitean. The left over constraints, i.e. the leading terms in the zero mode part of $\frac{1}{\alpha'}\left(X^2 - R^2\right)$ and in the oscillatory part of XP, can therefore be regarded as gauge fixing conditions.

To prove that the above choice of constraints is first class, we have to exhibit the closure of the constraint algebra generated by the Ls and the Ds. The algebra can be checked to be given by

$$
[L_0, D_0] = -2iL_0 \,, \qquad [L_0, D_p] = 0 \,, \qquad [L_n, D_0] = -i(L_n - L^*_n) \,, \tag{3.4}
$$

$$
[L_n, D_p] = \left(\sqrt{\frac{p+n}{n}} - \sqrt{\frac{n}{p+n}}\right) (L_{p+n} + L^*_{p+n}) + \zeta_{n,p}((L_{|p-n|} + L^*_{|p-n|}) \,, \tag{3.5}
$$

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where $\zeta_{n,p} = \frac{p}{\sqrt{n|n-p|}}$ if $p \neq n$ and $\zeta_{p,p} = 0$. Moreover[7] we have

$$[D_0, D_p] = -2\left(D_p - \frac{d+1}{2}\left(1 + (-1)^p\right)\right) \quad \text{and} \quad [D_p, D_q] = 0\,. \tag{3.6}$$

Notice that quantum mechanically one has to consider the operator D_p to be defined up to an additional constant due to normal ordering ambiguity. This can be added to cancel the c-number term appearing in the first commutator in (3.6). We will fix the actual value of such a quantity together with the proper ghost contribution by building a nilpotent quantum BRST charge for the system under consideration.

3.1.2 The quantum BRST charge

In order to build the BRST charge of the system, we introduce the relative anticommuting ghosts c_n, c_n^* and c_0 and the anti-ghosts b_n, b_n^* and b_0 (normalized by $[c_0, b_0]_+ = 1$, $[c_m, b_n^*]_+ = \delta_{mn}$, $[c_n^*, b_m]_+ = \delta_{nm}$ and other anti-commutators vanishing) for the contracted Virasoro constraints L_n, L_n^* and L_0 as well as the hermitean ghosts c'_p and c'_0 and the relative hermitean anti-ghosts b'_p and b'_0 (normalized by $[c'_0, b'_0]_+ = 1$, $[c'_n, b'_m]_+ = \delta_{nm}$ and other anti-commutators vanishing) for the contracted geometrical embedding constraints D_p and D_0. The ordering prescription that we keep for hermitean ghosts is the usual anti-symmetrization.

As it results from [2, 6], the BRST charge corresponding to the tensionless open string constraints in R^{d+1} is given by

$$\mathcal{Q}_{\text{open},R^{d+1}} = \frac{1}{2}c_0 L_0 + \sum_{n>0}\left[L_n^* c_n + c_n^* L_n - 2c_n^* c_n b_0\right]$$

and keeps into account the contracted Virasoro constraints only. The full BRST charge implementing the above full constrained system is obtained then by adding the geometric constraints as an improvement. The way we find natural to proceed is to notice that, since the algebra (3.4)–(3.5) is of the form $[L, D] = L$, then the geometric constraints D's can be promoted to operators commuting with $\mathcal{Q}_{\text{open},R^{d+1}}$ by adding suitable (bc) ghost terms.

Let us therefore introduce the following invariant bilinear combinations

$$l_{(mn)} = a_m \cdot a_n - c_m b_n - c_n b_m \qquad l^*_{(mn)} = a^*_m \cdot a^*_n + c^*_m b^*_n + c^*_n b^*_m \tag{3.7}$$

$$h_{mn} = a^*_m \cdot a_n + c^*_m b_n + b^*_m c_n + \frac{d-1}{2}\delta_{mn} \tag{3.8}$$

and[8] let us define the ghost completition of the contracted geometric constraints as

$$\Delta_0 = \frac{1}{2}(xp + px) - i[c_0, b_0] + i\sum_{m>0}(b^*_m c_m - c^*_m b_m) + \frac{1}{2i}\sum_{n>0}\left(l_{(nn)} - l^*_{(nn)}\right)$$

[7] To obtain the following commutation relations the reader might start from the commutation relations

$$[XX'(\sigma), XX'(\sigma')] = 0 \qquad \text{and} \qquad \left[\int PX, XX'(\sigma)\right] = -2XX'(\sigma)\,,$$

pass to the normal ordered expressions and then consider the leading order terms.

[8] The above family of quadratic combinations close to form the following $sp(\infty)$ algebra

$$[l_{(mn)}, l_{(pq)}] = 0 \qquad [l_{(mn)}, h_{pq}] = \delta_{np} l_{(mq)} + \delta_{mp} l_{(nq)} \tag{3.9}$$

and

$$\Delta_p = \sum_{n=1}^{p-1} \sqrt{\frac{p-n}{n}} \left(l_{(n,p-n)} + l^*_{(n,p-n)} + h_{n,p-n} + h_{p-n,n} \right) +$$
$$+ \sum_{n>0} \left(\sqrt{\frac{p+n}{n}} - \sqrt{\frac{n}{p+n}} \right) \left(l_{(n,p+n)} + l^*_{(n,p+n)} + h_{n,p+n} + h_{p+n,n} \right).$$

The commutation algebra satisfied by the above operators is

$$[\Delta_0, \Delta_p] = -2\Delta_p \qquad \text{and} \qquad [\Delta_p, \Delta_q] = 0\,. \tag{3.13}$$

We have now a clear synthetic framework to obtain the full quantum BRST charge

$$\mathcal{Q}_{\text{open},AdS} = \mathcal{Q}_{\text{open},R^{d+1}} + c_0'\Delta_0 + \sum_{p>0} c_p'\Delta_p + c_0' \sum_{p>0} [c_p', b_p']\,.$$

Notice that since the ghost completed geometrical constraints Δs commute with the unconstrained BRST charge $\mathcal{Q}_{\text{open},R^{d+1}}$ and fulfill the algebra (3.13) we have

$$\mathcal{Q}^2_{\text{open},AdS} = 0$$

irrespectively to the value of the space-time dimension. This shows that, as in the flat case, also on AdS space the whole conformal anomaly scales away and the problem of critical dimension does not exist anymore in the tensionless limit.

Moreover, since the expressions we started from were invariant under the AdS rotational group $\mathrm{SO}(2, d-1)$, also the tensionless limit is. More concretely, one can check that the $\mathrm{SO}(2, d-1)$ generators

$$J_{\mu\nu} = \frac{1}{2}(x_\mu p_\nu - x_\nu p_\mu) - \frac{i}{2} \sum_{n>0} (a^*_{n\mu} a_{n\nu} - a^*_{n\nu} a_{n\mu}) \tag{3.14}$$

are such that $[J_{\mu\nu}, \mathcal{Q}_{\text{open},AdS}] = 0$.

3.2 Closed strings

In order to treat the closed string case, we follow a procedure analogous to the one we developed in the open string case. Let us expand the σ-model canonical coordinates in oscillators, that is (now $\sigma \in [0, 2\pi]$)

$$X^\mu(\sigma) = x^\mu + \sum_{n>0} \sqrt{\frac{\alpha'}{2n}} \left(a^\mu_n e^{-in\sigma} + \bar{a}^\mu_n e^{in\sigma} + \text{h.c.} \right)$$
$$P^\mu(\sigma) = \frac{p^\mu}{2\pi} + \frac{1}{2\pi} \sum_{n>0} \sqrt{\frac{n}{2\alpha'}} \left(-ia^\mu_n e^{-in\sigma} - i\bar{a}^\mu_n e^{in\sigma} + \text{h.c.} \right),$$

$$[h_{mn}, h_{pq}] = \delta_{np} h_{mq} - \delta_{mq} h_{pn} \tag{3.10}$$
$$[l^*_{(mn)}, l^*_{(pq)}] = 0 \qquad [l^*_{(mn)}, h_{pq}] = -\delta_{nq} l^*_{(mp)} - \delta_{mq} l^*_{(np)} \tag{3.11}$$
$$[l_{(mn)}, l^*_{(pq)}] = h_{qm}\delta_{np} + h_{pm}\delta_{nq} + h_{qn}\delta_{mp} + h_{pn}\delta_{mq} \tag{3.12}$$

which is useful to check the algebraic calculations. Notice that all these bilinears commute with the unconstrained BRST charge $\mathcal{Q}_{\text{open},R^{d+1}}$.

where the above modes satisfy the CCRs

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \qquad [a_n^\mu, (a^*)_m^\nu] = \eta^{\mu\nu}\delta_{nm} \qquad [\bar{a}_n^\mu, (\bar{a}^*)_m^\nu] = \eta^{\mu\nu}\delta_{nm} \tag{3.15}$$

and the other commutators vanishing.

Calculating the Virasoro constraints and performing the leading order terms extraction as we did in the open string case, we find that the left over constraints for the tensionless closed string are

$$\begin{aligned} &p^2\,, \qquad L_n = p\cdot a_n\,, \qquad L_n^* = p\cdot a_n^*\,,\\ &\bar{L}_n = p\cdot \bar{a}_n\,, \qquad \bar{L}_n^* = p\cdot \bar{a}_n^*\,, \qquad N-\bar{N} = \sum_{n>0} n\left(a_n^*\cdot a_n - \bar{a}_n^*\cdot \bar{a}_n\right) = 0\,, \end{aligned}$$

the last one being the usual level matching condition (and we don't define any $L_0 = \bar{L}_0 = p^2$, but we just keep denoting p^2 not to cause extra confusion with the level matching and the notation which makes sense within the tensile string usual notation).

As far as the geometric constraints

$$\frac{1}{\alpha'}\left(X^2 - R^2\right) \qquad \text{and} \qquad XP$$

are concerned, one can work out easily their leading order factors in the tensionless limit and, as in the open free string case, single out the oscillatory part of the first and the zero mode part of the second, that is the leading order of $\frac{1}{\alpha'} : XX' : (\sigma)$ and $\oint : XP :$.

Calculating the relevant Fourier modes and leading terms, we have $\frac{1}{\alpha'} : XX' : (\sigma) = \frac{i}{2}\sum_{p>0}\left(C_p e^{ip\sigma} - C_p^* e^{-ip\sigma}\right) + o\left(\frac{1}{\sqrt{\alpha'}}\right)$, where

$$\begin{aligned} C_p = &\sum_{m=1}^{p-1}\sqrt{\frac{m}{p-m}}\left(\bar{a}_{p-m} + a_{p-m}^*\right)\cdot\left(\bar{a}_m + a_m^*\right) + \\ &+\sum_{n>0}\left(\sqrt{\frac{p+n}{n}} - \sqrt{\frac{n}{p+n}}\right)\left(a_n + \bar{a}_n^*\right)\cdot\left(\bar{a}_{p+n} + a_{p+n}^*\right) \end{aligned} \tag{3.16}$$

and

$$C_0 = \oint : XP := \frac{1}{2}(xp + px) + i\sum_{n>0}\left(\bar{a}_n^* a_n^* - \bar{a}_n a_n\right).$$

Notice that C_0 is hermitean, while C_p is conjugated to C_p^* and viceversa. (We don't explicitly write the normal ordered expression for C_p since it coincides with the one we gave above).

It is straightforward to verify that the algebra of the constraint functions closes to a Lie algebra and that therefore the BRST quantization procedure can be developed as in the open string case. This shows that also in the closed string case it is possible to single out a subset of the geometrical constraints in order to present the constrained hamiltonian system just in terms of first class constraints only. Specifically, the actual form of the constraint algebra is given by the following commutation relations

$$
\begin{aligned}
[L_n, L^*_m] &= p^2\delta_{nm}, \qquad [\bar{L}_n, \bar{L}^*_m] = p^2\delta_{nm}\,,\\
[L_n, N-\bar{N}] &= nL_n\,, \qquad [\bar{L}_n, N-\bar{N}] = -n\bar{L}_n\,,\\
[p^2, C_0] &= -2ip^2\,, \qquad [L_n, C_0] = -iL_n + i\bar{L}^*_n\,, \qquad [\bar{L}_n, C_0] = -i\bar{L}_n + iL^*_n\,,\\
[L_n, C_p] &= \left(\sqrt{\frac{n}{n-p}} - \sqrt{\frac{n-p}{n}}\right)(L_{n-p} + \bar{L}^*_{n-p}) \quad \text{if} \quad n>p\\
[L_n, C_p] &= \left(\sqrt{\frac{p-n}{n}} + \sqrt{\frac{n}{p-n}}\right)(\bar{L}_{p-n} + L^*_{p-n}) \quad \text{if} \quad n<p\\
[L_p, C_p] &= 0 \qquad [\bar{L}^*_n, C_p] = [L_n, C_p]\\
[\bar{L}_n, C_p] &= [L^*_n, C_p] = \left(\sqrt{\frac{p+n}{n}} - \sqrt{\frac{n}{p+n}}\right)(\bar{L}_{n+p} + L^*_{n+p})\\
[C_0, C_p] &= -2C_p\,,
\end{aligned}
$$

the others being vanishing or can be obtained by hermitean conjugation of the above ones. Notice that the above algebra is not a direct product of two copies of the open one. Here, in fact, we see explicitly how the curvature of the background affects the left/right sectors mixing (as it is expected to happen from a more general point of view).

Also the construction of the quantum BRST charge follows a path similar to the open string case. Introducing the ghosts for the geometric constraints c'_p, c'_0 and $(c')^*_p$ as well as the conjugated anti-ghosts $(b')^*_p$, b'_0 and b'_p, the BRST charge can be built as

$$
\mathcal{Q}_{\text{closed},AdS} = \mathcal{Q}_{\text{closed},R^{d+1}} + \sum_{p>0}\left((c')^*_p\Gamma_p + \Gamma^*_p c'_p\right) + c'_0\Gamma_0 + [[b'c'c']]\,, \tag{3.17}
$$

where

$$
\begin{aligned}
\mathcal{Q}_{\text{closed},R^{d+1}} = {} & \frac{1}{2}c_0p^2 + \sum_{n>0}(c^*_n p a_n + p a^*_n c_n + \bar{c}^*_n p\bar{a}_n + p\bar{a}^*_n\bar{c}_n) + \\
& + \hat{c}_0\sum_{n>0} n\left(a^*_n a_n + c^*_n b_n + b^*_n c_n - \bar{a}^*_n\bar{a}_n - \bar{c}^*_n\bar{b}_n - \bar{b}^*_n\bar{c}_n\right) - \\
& - 2b_0\sum_{n>0} n\,(c^*_n c_n + \bar{c}^*_n\bar{c}_n)\,,
\end{aligned}
$$

$\Gamma_p = C_p + [\text{ghosts}]$ are the ghost completition of the constraints C_p such that $[\mathcal{Q}_{\text{closed}\,R^{d+1}}, \Gamma_p] = 0$ and still satisfy the internal algebra of the constraints, namely

$$
[\Gamma_0, \Gamma_p] = -2\Gamma_p\,, \qquad [\Gamma_0, \Gamma^*_p] = 2\Gamma^*_p \tag{3.18}
$$

and the others vanishing. Finally, the term $[[b'c'c']]$ is built from the algebra (3.18) structure constants and is equal to $2c'_0\sum_{p>0}\left((b')^*_p c'_p + (c')^*_p b'_p\right)$.

Since the algebra (3.18) is fulfilled and the Γs commute with $\mathcal{Q}_{\text{closed},R^{d+1}}$, then the BRST charge (3.17) satisfies

$$
\mathcal{Q}^2_{\text{closed},AdS} = 0
$$

identically.

Notice moreover that all the constraints commute with the generators of the AdS isometry group $\mathrm{SO}(2, d-1)$, i.e. with

$$J_{\mu\nu} = \frac{1}{2}\left(p_\mu x_\nu - p_\nu x_\mu\right) + \frac{i}{2}\sum_n \left(a^*_{n\mu}a_{n\nu} - a^*_{n\nu}a_{n\mu} + \bar{a}^*_{n\mu}\bar{a}_{n\nu} - \bar{a}^*_{n\nu}\bar{a}_{n\mu}\right)$$

which therefore commute with the BRST charge too.

4. Conclusions and open questions

In this short note we have studied few basic properties of tensionless bosonic strings in AdS space-time, namely the very existence of a covariant quantization scheme for such a system. We found that the special simplification in the constraint algebra that takes place in such a limit enables an explicitly covariant quantization scheme of the theory in any spacetime dimension. Let us notice that the light-cone gauge fixed approach developed in [14], although seemingly not rigorous, indirectly suggests such a result.[9]

In principle, the results that we obtained can be extended to any spacetime which can be obtained as a quadric in a higher dimensional flat space, as dS space for example. The physical difference between the positive and negatively curved space have to be understood from the explicit calculation of the spectrum of the theories, that is from the calculation of the BRST state cohomology whose structure naturally depends on the signature of the defining quadratic form η.

The second quantization of the free tensionless string theories that we just developed results in an infinite chain of higher spin theories on AdS (about this subject see [17]–[21]). In such a general framework it is possible indeed to study also the interaction of tensionless strings as already proposed in [6]. Actually, we can extend at some rate the arguments about string fragmentation and world-sheet picture instability by [22] which indicate that the first quantized interacting string gets literally undone in the tensionless limit. This phenomenon has a clear analogous in QED which is the IR-catastrophe in first quantization when finite energy amounts can be emitted in the form of an infinity of soft photons. Notice that some caveat has to be raised here, since perturbative IR fluctuations are typically dumped in negatively curved space-times [23] and therefore the analogous of the analysis in [22] in AdS space should be carried out carefully possibly giving a tendential dumping of the string fragmentation effect. Such an unclear picture promotes the string field approach to be the natural framework to correctly formulate tensionless strings interactions. In order to do this, one has to extend the usual string field theory techniques to AdS to build the three string vertex [24, 25]. The value of the AdS radius is expected to play a role in the interacting theory due to the deformation of the gauge symmetry.

[9] Notice that a different approach to the tensionless string in AdS appeared also in [15]. That approach is a limiting case of the construction carried out in [16] and is based on a coset WZNW realization of the AdS σ-model which therefore implies the presence of a specific balancing background antisymmetric field. It follows that such a approach refers to a different set up with respect to the one we have been considering in this paper.

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The approach we developed could in fact be extended to superstring theories too with results naturally much similar to the ones we have found for the bosonic case. In particular one could consider the tensionless limit of type IIB string on $AdS_5 \times S^5$ (which can be easily described as a product of two quadrics in a twelve dimensional space) in order to get some new inputs for AdS/CFT in the tensionless limit [26]–[28]. The second quantization of the tensionless string theories that we obtained here could in fact shed some light on that subject. Actually, to advance in such a direction one should study the spectrum of the tensionless theory, that is (from our perspective) one has to solve for the state cohomology of the BRST charge $\mathcal{Q}_{\text{open},AdS}$ or $\mathcal{Q}_{\text{closed},AdS}$, and compare it with the spectrum of conserved currents of a proper boundary theory. The actual calculation of the BRST state cohomology is obviously the first development to be carried out. It is clear that this will classify the physical states in terms of IURs of the AdS group with a spectrum which we expect highly symmetric and irreducible to a deformation of the flat space string spectrum, as the results in [19] already suggest.

These and other possible aspects of the tensionless limit of string theories are open issues for further research.

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